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LA-UR--87-2314

DE87 013149

TITLE **MODES, MASSES, METALLICITES, AND MAGNITUDES OF RR LYRAE VARIABLES**

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SUBMITTED TO **IAU Colloquium 95, June 1987**
Tucson, Arizona

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7/1/87

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MASTER

MODES, MASSES, METALLICITIES, AND MAGNITUDES OF RR LYRAE VARIABLES

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The observed pulsation modes for RR Lyrae variables have long been understood to be the radial fundamental for the Bailey ab type and the radial first overtone for the c type variables (Schwarzschild, 1941). Six years ago many double-mode RR Lyrae (RRd) variables were found in globular clusters such as M15 and M3 by Cox, Hodson, and Clancy (CHC, 1983), supplementing the single known field double-mode RR Lyrae variable AQ Leonis. About three dozen are now known in population II systems due to mostly the work of Nemec and collaborators. See Clement et al. (1987) for the latest summary. Most of this review involves these double-mode RR Lyrae variables, because they can tell us a great deal about population II stars.

The diagram of the period ratio versus period was first exploited for determining masses by Jorgensen and Petersen (1967) and Petersen (1973). Figure 1 gives this Petersen diagram for the RR Lyrae variables for the period, luminosity, and T_{eff} range of the double-mode RR Lyrae variables. Also plotted are the periods and period ratios for all the known double-mode RR Lyrae variables in the several population II systems and for the only confirmed field variable AQ Leo. The lines of constant mass vary somewhat with luminosity, T_{eff}, and composition, making it important that we construct the diagram for the appropriate conditions if it is to be used for determining masses. The M15 globular cluster variables indicate a mass of almost exactly $0.65M_{\odot}$.

Using the mass of $0.65M_{\odot}$ and the King Ia composition (Cox and Tabor, 1976), we construct the theoretical Hertzsprung-Russell diagram of Figure 2. The expression for the period as a function of the mass, luminosity, and effective temperature (actually the accurate representation of the period-mean density relation) is given in Table 1 for various compositions including a widely used fit suggested by van Albada and Baker (1971). This latter fit is not specifically designed for the double-mode RR Lyrae variables, and it should not be used in quantitative arguments. My fit expressions match the individually calculated theoretical periods always to better than 0.01 in log P, and for some compositions better than 0.005. Lines of constant period and constant period ratio are indicated on the H-R diagram. Blue edges for the overtone and fundamental modes are drawn at their positions which recently have been shown (Stellingwerf 1984) to be insensitive to the helium content.

The M15 variables observed by Sandage, Katem, and Sandage (SKS, 1981) are put on this diagram by selecting a tentative distance modulus, assigning them absolute luminosities, entering the plot at these luminosities, and moving horizontally to the lines for the observed periods. If the luminosities are too large, then the stars will be plotted too blue to fit into the observed (and also theoretical) instability strip. Otherwise, if the absolute luminosities are taken as too small, the stars will be plotted much too red. Note that if we had used the Stellingwerf (1975ab) fit for the King Ia composition, at $0.65M_{\odot}$, the best fit into the instability strip would be about 0.01 in $\log T_{\rm eff}$ bluer and about 0.04 fainter in $\log L/L_{\odot}$ to remain in the instability strip. The luminosity of the horizontal branch can then be determined to be about $\log L/L_{\odot}=1.78\pm0.04$.

It was noted in the first work on this problem (CHC, 1983) that this procedure gives the double-mode stars all at very closely the same effective temperature of about 7000K ($\log T_{\rm eff}=3.845$). They are also located near to their theoretically expected position just redward of the fundamental mode blue edge. If the observed colors by SKS or those of Bingham et al. (1984) are used together with the period, the close clustering of the double-mode variables on the H-R diagram is not seen, because the colors averaged over the pulsation cycle and then converted to effective temperatures are not highly accurate.

We know that there needs to be a small variation in mass from star to star to get the large observed color width of the horizontal branch (Rood, 1973), and therefore the assumption of all stars being exactly $0.65M_{\odot}$ needs to be relaxed. An increase or decrease of 0.05 $M_{\odot}$ would involve horizontal movement on the diagram as indicated, but probably the double-mode variables masses vary by a lesser amount as seen in Figure 1. The mass scatter is known theoretically to affect the total width of the horizontal branch (lower masses at higher $T_{\rm eff}$), and the RR Lyrae variables occupy only the central part.

The discovery of the double-mode RR Lyrae variables in population II systems then has clarified our knowledge not only about their masses, but also their absolute luminosities, effective temperatures, and helium contents. If a Y value of 0.199 (King IIa) instead of 0.299 (King Ia) had been used, then, for fixed period and $T_{\rm eff}$, the logarithm of the luminosity would have been $0.035/0.83=0.042$ or about 0.1 magnitude lower, using the constants for these two material property tables given in Table 1. The lower helium directly lowers the luminosity.

Actually, a lower luminosity by maybe only 0.02 in log L, corresponding to Y=0.25, conflicts with evolution theory (Sweigart, Renzini, and Tornambe, 1987) that at this Y and mass puts the zero age horizontal branch at log L of less than 1.70. Pulsation theory coupled with evolution theory and the observations of the variable stars then gives high masses, high helium, and high luminosities for the Oosterhoff type II cluster M15 stars.

For M3, the mass of the two double-mode variables is $0.55 M_\odot$ (CHC), and the horizontal branch luminosity is log $L/L_\odot$ =1.66. The transition temperature between the overtone and fundamental pulsators is still at about 7000K. For the low mass and luminosity of M3, evolution theory is again in difficulty. Only if Z=0.01, a very high value for a population II system, can stars be predicted in the instability strip.

A few words should be said about the Sandage (1982) effect. At a given $T_{\rm eff}$, Sandage found that

$$\Delta \log P_0 = -0.116 \Delta [\mathrm{Fe/H}],$$

and the coefficient is well illustrated by the differences in the periods in M3 and M15. M15 has longer periods, but lower [Fe/H] by 0.5. Other recent values of the coefficient tend to be smaller, such as -0.085 by Bell (1984), -0.094 by Bingham et al. (1984), and -0.084 by Lub (1987). I can agree at least with the $\Delta \log P_0$, because using any of the fits in Table 1 at a fixed $T_{\rm eff}$, the greater M15 luminosity, compared to M3, overcomes a period decrease due to the larger M15 masses. The fits give a change in log P of 0.052, and when the metallicity change is divided into this number, the Sandage coefficient is obtained. The question remaining is where is the Sandage effect in ω Cen where in a single cluster there is a spread in metallicity without an obvious accompanying period shift.

I have recently wondered whether the Δ[Fe/H] as measured by spectroscopic analyses of globular cluster horizontal branch and red giant stars (see Zinn and West, 1984) is really accurate. My motivation is that Sweigart, Renzini and Tornambe (1987) can get a small Sandage effect, due mostly to the difference between evolution tracks after the zero age horizontal branch, but they cannot get the coefficient more negative than about -0.04. Maybe the effects of element separation (winds?) in the atmosphere of these stars operates to confuse the quantitative abundance analyses. We do know that the width of the red giant branch in ω Cen indicates a metallicity spread about as seen among the RR Lyrae variables by Butler, Dickens and Epps (1978), but I suggest that these RR Lyrae variables may display a metallicity wrong by 0.5 or more in the logarithm. That is, the interior metallicity of M15 is lower by 0.5 or more than seen on the surface, and M3 metallicity in the interior is close to Z=0.01.

We now consider the difficult problem of what causes double-mode pulsations. Several groups have studied hydrodynamic representations of the RR Lyrae pulsations, and have failed to get permanently mixed mode behavior (Hodson and Cox, 1982). Now Ostlie, working at Los Alamos and Weber State College, has probably found the key ingredients: time dependent convection with a high helium abundance.

Figure 3 sets the background for the double-mode behavior discussion. Here growth rate is plotted versus $T_{\rm eff}$. The two upper lines give the growth rates of the fundamental and overtone modes starting from static, equilibrium models. The intersection of these lines with the zero axis indicates the blue edge in $T_{\rm eff}$ where, blueward, models are stable against pulsations. As observed, the overtone blue edge (1HBE) is bluer than the fundamental blue edge, giving a region of about $\Delta \log T_{\rm eff}$ of 0.02 where only overtone (Baily type c) pulsators occur.

The other two solid lines give the most recent previous calculations for the stability of the overtone in the presence of the full amplitude fundamental, and the stability of the fundamental in the environment of the overtone at full amplitude (Simon and Cox, unpublished). The 1H in F line shows that at the fundamental blue edge (FBE), the stability in the presence of the full amplitude fundamental is the same as for a static model because there the fundamental is stable against pulsation, and therefore models are indeed static. Likewise, at the 1HBE, the stability of the fundamental in the overtone (F in 1H) is the same as for a static model. Clearly the transition line (TL) between the overtone and fundamental pulsators in the H-R diagram at various luminosities (only one luminosity cut is given here), is slightly redder than the FBE. Ostlie has shown that in the small region at just less than $\log T_{\rm eff}$=3.85, the dashed lines obtain, and the transition line is not sharp. A star evolving blueward would start double-mode behavior at $\log T_{\rm eff}$ less than 3.85 and continue to $T_{\rm eff}$ greater than $\log T_{\rm eff}$ 3.85. Redward evolution would have the same limits.

What is the required helium abundance for this double-mode occurrence? Observations indicate that mass and luminosity are not limitations, because both the low metallicity, high luminosity, high mass M15 stars and the high metallicity, low luminosity, low mass M3 stars show double-mode behavior. Perhaps the lack of double-mode pulsators in ωCen and M5 and the scarcity of them in M3 may simply be due to a helium abundance that is too small.

Stellingwerf (1974,1975a) has shown how to diagnose the possibility of two simultaneous modes in RR Lyrae models. Figure 4 gives the result of the procedure. Here the work each cycle to promote the fundamental mode in the presence of the overtone (not at pure mode full amplitude, however) for each mass shell of the 60 zone $0.65 M_\odot$, 7000 K, $\log L/L_\odot=1.78$, Z=0.001 model (M15 RR Lyrae variable conditions) is plotted. Integration of this plot shows that there is slightly more positive work than negative, and the mode will tend to grow in kinetic energy amplitude. Especially interesting is that the surface hydrogen driving (by the classical κ and γ effects) is small compared to the helium driving between zones 38 and 53 as well as the Stellingwerf (1978) "bump" helium driving between zones 27 and 30. Thus more helium gives more helium driving at the expense of hydrogen driving, but helium is more efficient in promoting the mode switch from 1H to F.

Figure 5 shows the same work plot for the growth of the overtone in the same model at the same amplitude. Here the overtone that is being studied does not have so much amplitude at deep zones near 30, and the Stellingwerf "bump" does not appear. Again integration of the driving and damping gives slow mode kinetic energy growth.

The role of time dependent convection seems crucial. Inspection of the timing of the convective luminosity shows that convection adds to the total outward luminosity at the start of the expansion phase, to reinforce pulsations.

Recent work by Kovacs and me has been done to clarify the Petersen diagram accuracy. Figure 6 gives the Petersen diagram he published (Kovacs, 1985). It shows that in the region of 0.5 day for the fundamental mode period, the CHC line for 0.55 $M_\odot$ is that for just under $0.65 M_\odot$, and for the $0.65 M_\odot$ line, Kovacs gets about $0.75 M_\odot$. The cause of the difference may be attributed to different models or different pulsation calculations. However, using the Los Alamos programs, we both have obtained period solutions that verify the lines of Figure 1. Tables 2 and 3 give representations for these lines where the period ratio fits reproduce the ratios to better than 0.002 in all cases including some conditions that are significantly different from the actual masses, luminosities and effective temperatures of the observed double-mode RR Lyrae variables.

Evolution theory would find it easier if the M15 masses were $0.75 M_\odot$ and the M3 masses $0.65 M_\odot$, because they can be matched by tracks with Y=0.30 and Z=0.001. However, the fits of Table 1 show that at larger mass the periods require a larger luminosity by $\Delta \log L/L_\odot = 0.79 \Delta \log M/M_\odot$. This makes the luminosity for the M15 horizontal branch 0.05 in $\log L$ brighter, probably in disagreement with observations.

Another somewhat independent test of the masses of the double-mode RR Lyrae variables can be made using the derived luminosities that fit the instability strip. Pulsation masses can be derived using the equations of Table 1 when periods, luminosities, and $T_{\rm eff}$ values are known. Figure 7 gives these masses for the M15 variables with the horizontal branch luminosity given as $\log L/L_\odot = 1.78$ and the SKS $T_{\rm eff}$ values. Here we use the individual observed $T_{\rm eff}$ values rather than having only the total range fit into the instability strip. Since the mass changes with the $T_{\rm eff}$ to the 3.4/.65=5.2 power, one can probably move the reddest variables to the left and down and the bluest variables to the right and up. Nevertheless the mean for the fundamental mode variables is 0.71 $M_\odot$ and for the overtone variables 0.62 $M_\odot$. It appears that the masses average nearer to 0.65 $M_\odot$ than 0.7 $M_\odot$ or 0.6 $M_\odot$.

Figure 8 shows the same kind of data for M3. Again there is a large spread, but the mean mass is 0.53 $M_\odot$ for the fundamental pulsators and 0.55 $M_\odot$ for the overtone ones.

It must be pointed out that if the larger Kovacs masses, by some factor, are more nearly correct, the pulsation masses will also be calculated to be exactly the same factor larger. An argument for these larger masses is that after further asymptotic giant branch evolution, these same stars will appear as pre-white dwarfs.

The period spacing for the g modes in PG 1159-035 indicate a mass of $0.60M_{\odot}$ (Kawaler 1987), with $0.55M_{\odot}$ not easiiy matched. The observed mass loss on the AGB must reduce the mass somewhat, making the $0.55M_{\odot}$ unlikely.

I conclude with the uncertainty in RR Lyrae variable masses and luminosities, but with a certainty that they have large helium content. The M3 masses are indeed lower than in M15 as Stobie (1971) suggested long ago. The period-mean density relation given by van Albada and Baker (1971) needs slight modification. Time dependent convection needs further study, but it probably is the explanation for double-mode behavior if the Y is large enough. The puzzles in ω Cen I interpret as a helium content of its variable stars not large enough to give double-mode variables, and its surface Z may not accurately represent its interior Z.

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Table 1
Period-Mean Density Coefficients
$\log P_0 = A + B \log L/L_\odot + C \log M/M_\odot + D \log T_{\rm eff}$

Composition	A	B	C	D
Stellingwerf Fit (Y=0.299 Z=0.001)	11.406	0.830	-0.659	-3.451
King Ia (Y=0.299 Z=0.001)	11.331	0.829	-0.655	-3.430
King Ia (without convection)	11.200	0.829	-0.654	-3.396
KingIIa (Y=0.199 Z-0.001)	11.366	0.828	-0.654	-3.439
van Albada and Baker	11.497	0.84	-0.68	-3.48

Table 2
Period Ratio Coefficients
$\log P_1/P_0 = a + b \log(L/L_\odot\text{-}1.703) + c \log(M/M_\odot/0.65) + d \log(T_{\rm eff}/7000)$

Composition	a	b	c	d
Stellingwerf Fit (Y=0.299 Z=0.001)	-0.1256	-0.0284	0.0526	0.0986
King Ia (Y=0.299 Z=0.001)	-0.1265	-0.0240	0.0438	0.0806
King Ia (without convection)	-0.1264	-0.0237	0.0427	0.0826
KingIIa (Y=0.199 Z=0.001)	-0.1261	-0.0250	0.0473	0.0990

Table 3
Period Ratio Versus Period Coefficients
$P_1/P_0 = \alpha + \beta\ (P_0(day) - 0.5) + \gamma\ (P_0(day) - 0.5)^2$

King Ia

$M/M_\odot$	α	β	γ
0.55	0.7437	-0.0357	-0.0036
0.65	0.7463	-0.0375	+0.0407
0.75	0.7488	-0.0391	+0.0790

King IIa

$M/M_\odot$	α	β	γ
0.55	0.7440	-0.0445	+0.0062
0.65	0.7469	-0.0431	+0.0345
0.75	0.7496	-0.0428	+0.0668

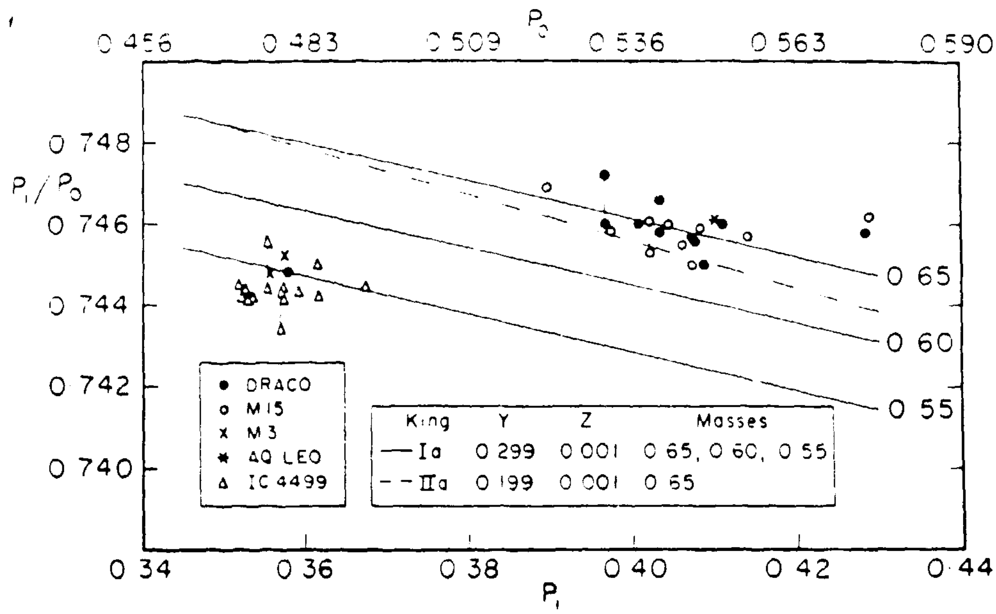

Figure 1. The Petersen diagram for RR Lyrae variables AQ Leo, and those in M3, M15, IC 4499, and Draco. The most recent calibration has the $0.65M_{\odot}$ lines for both compositions between the two lines plotted at the 0.55 day fundamental period.

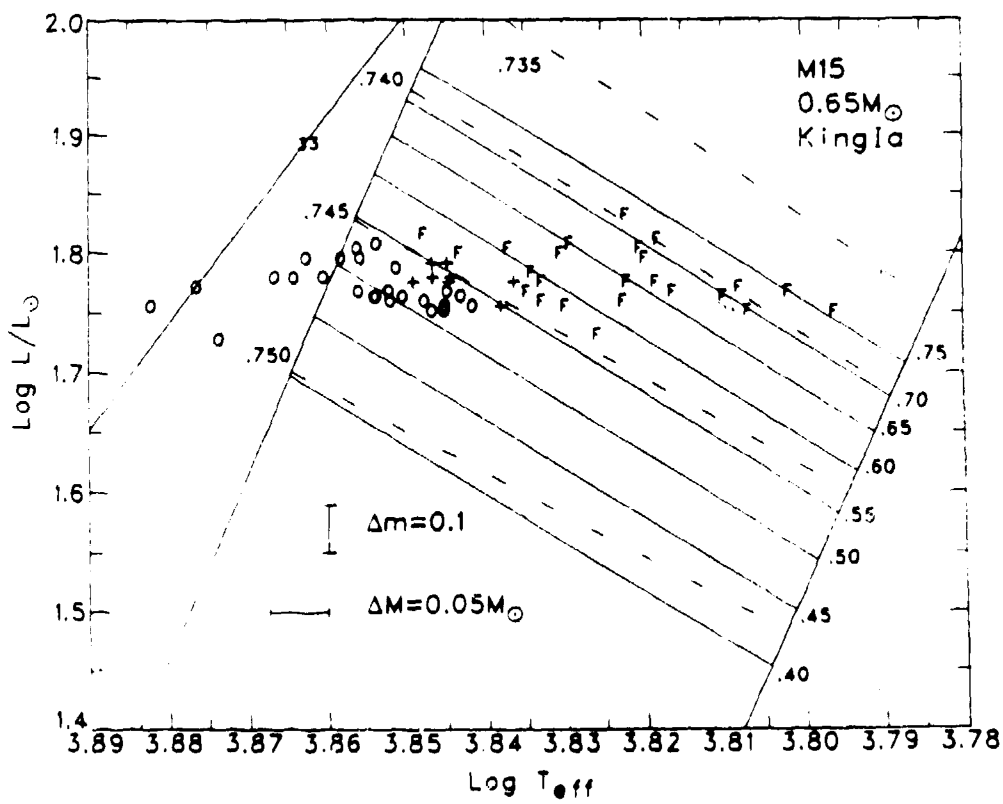

Figure 2. The theoretical H-R diagram for $0.65M_{\odot}$ and the King Ia composition with the M15 variables plotted at their observed periods and relative luminosities.

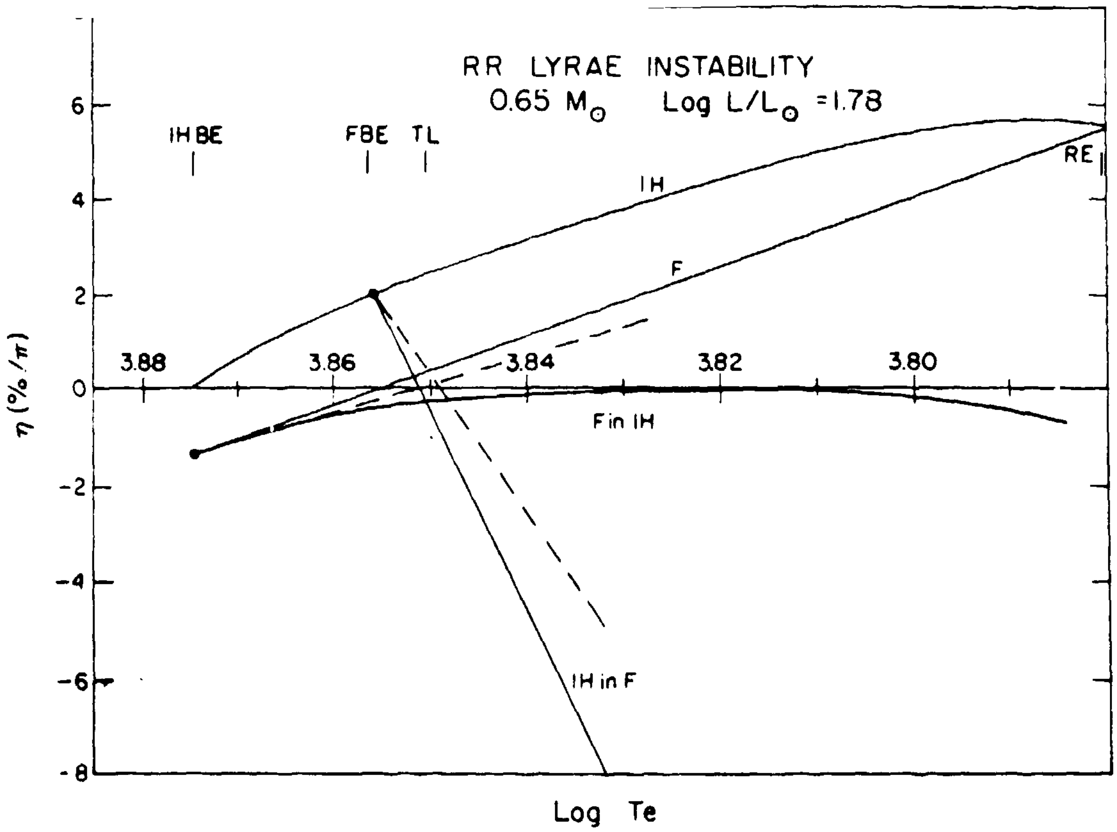

Figure 3. Growth rates for the fundamental and overtone modes in static and pulsating models.

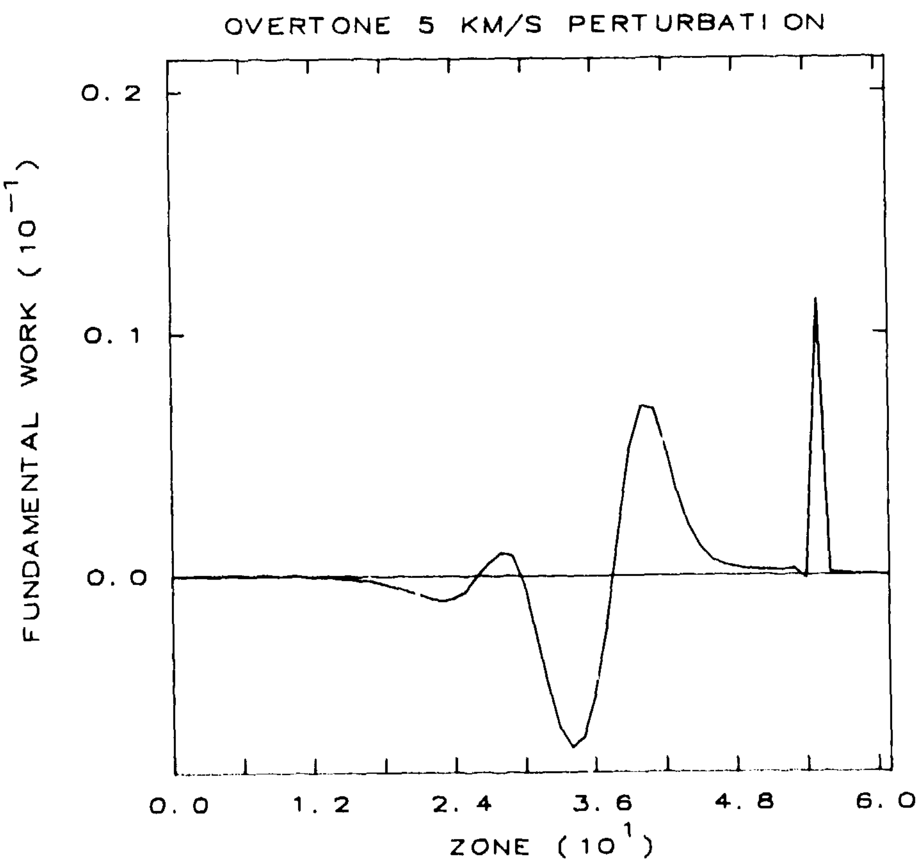

Figure 4. The work per zone in a pulsating model to promote the fundamental mode.

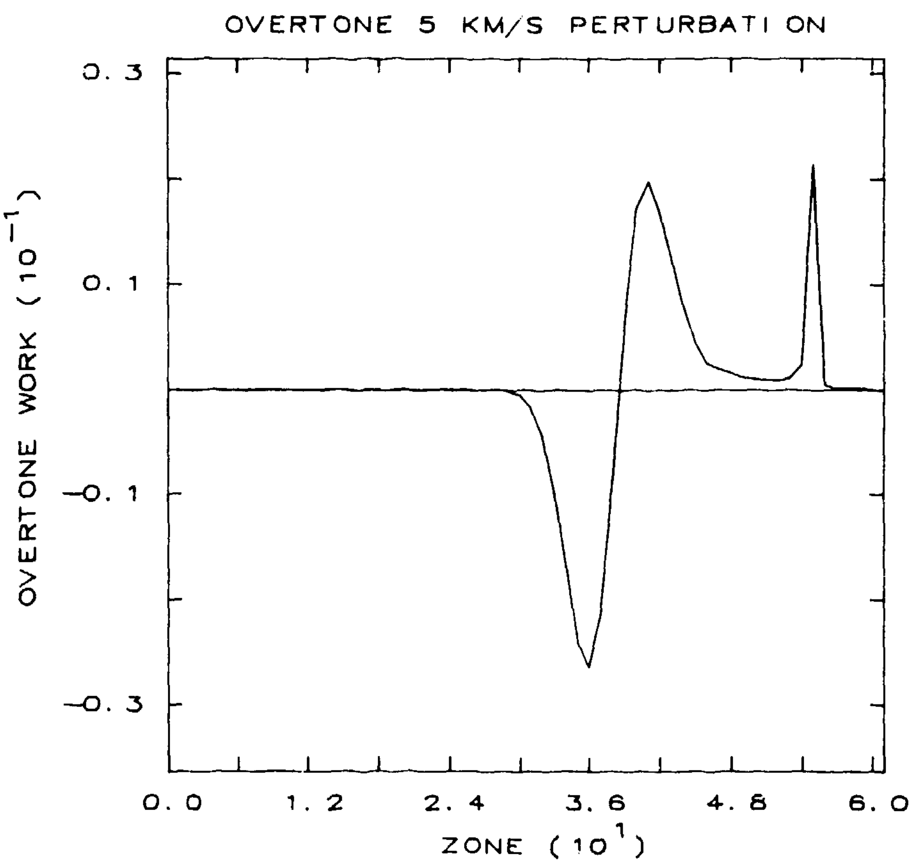

Figure 5. The work per zone in a pulsating model to promote the first overtone mode.

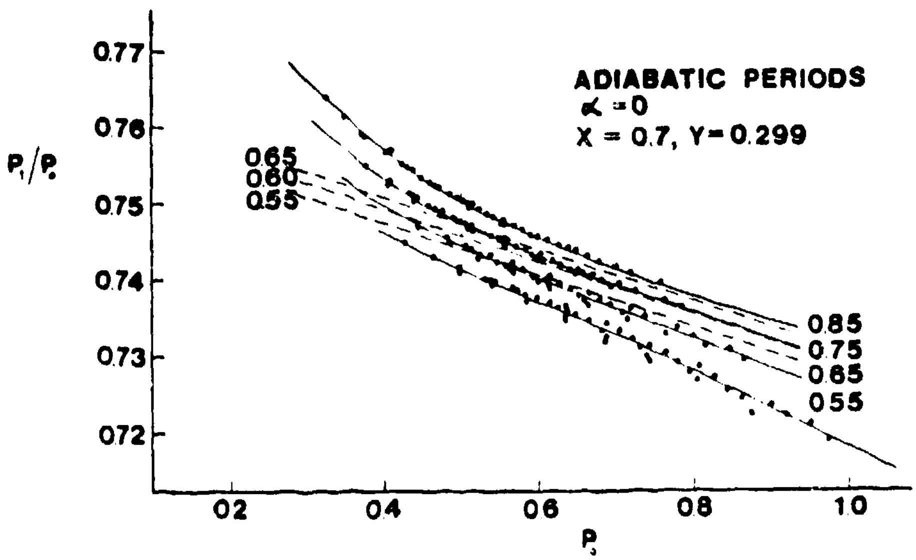

Figure 6. The Petersen diagram as published by Kovacs.

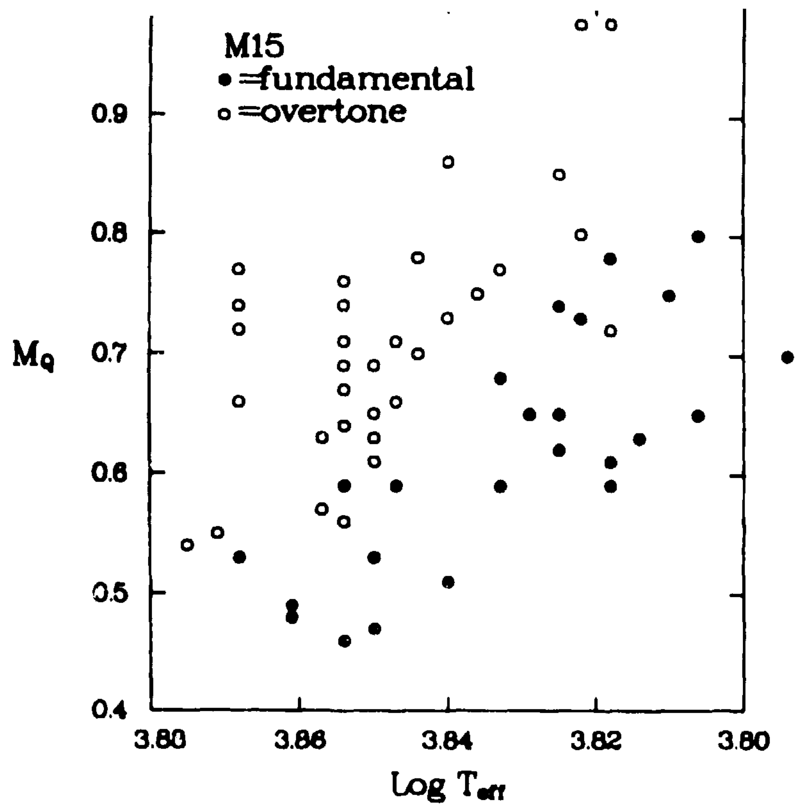

Figure 7. Pulsation masses for the M15 RR Lyrae variables using the SKS data plotted versus effective surface temperature.

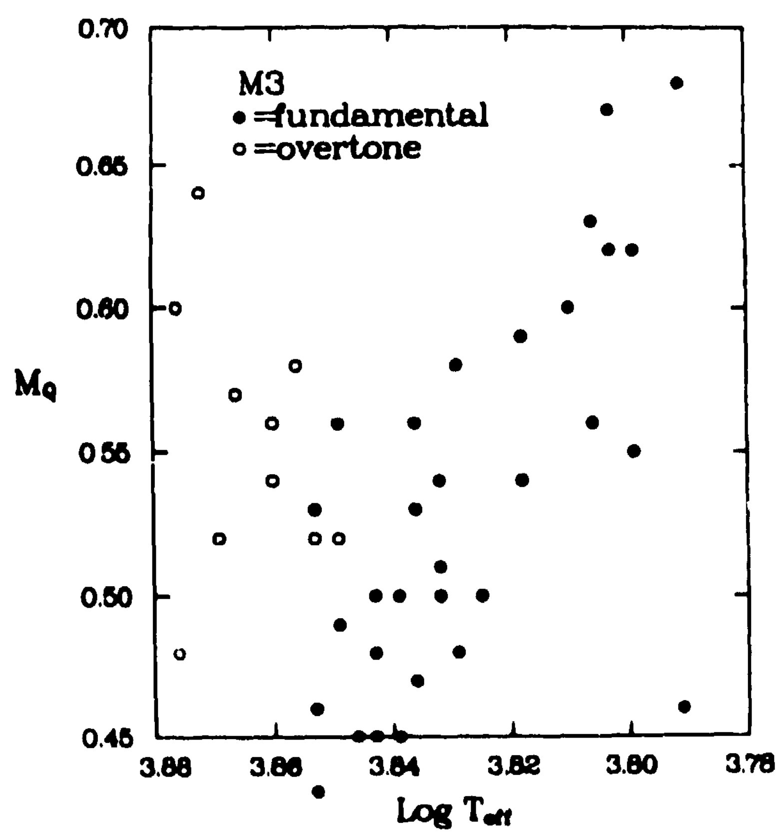

Figure 8. Pulsation masses for the M3 RR Lyrae variables using the SKS data plotted versus effective surface temperature.